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Uncertainty and Sensitivity Analysis in Aircraft Operating Costs in Structural Design Optimization

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The paper focuses on the development of an aircraft design optimization methodology that models uncertainty and sensitivity analysis in the tradeoff between manufacturing cost, structural requirements, and aircraft direct operating cost. Specifically, rather than only looking at manufacturing cost, direct operating cost is also considered in terms of the impact of weight on fuel burn, in addition to the acquisition cost to be borne by the operator. Ultimately, there is a tradeoff between driving design according to minimal weight and driving it according to reduced manufacturing cost. The analysis of cost is facilitated with a genetic-causal cost-modeling methodology, and the structural analysis is driven by numerical expressions of appropriate failure modes that use ESDU International reference data. However, a key contribution of the paper is to investigate the modeling of uncertainty and to perform a sensitivity analysis to investigate the robustness of the optimization methodology. Stochastic distributions are used to characterize manufacturing cost distributions, and Monte Carlo analysis is performed in modeling the impact of uncertainty on the cost modeling. The results are then used in a sensitivity analysis that incorporates the optimization methodology. In addition to investigating manufacturing cost variance, the sensitivity of the optimization to fuel burn cost and structural loading are also investigated. It is found that the consideration of manufacturing cost does make an impact and results in a different optimal design configuration from that delivered by the minimal-weight method. However, it was shown that at lower applied loads there is a threshold fuel burn cost at which the optimization process needs to reduce weight, and this threshold decreases with increasing load. The new optimal solution results in lower direct operating cost with a predicted savings of $640/m^2$ of fuselage skin over the life, relating to a rough order-of-magnitude direct operating cost savings of \$500,000 for the fuselage alone of a small regional jet. Moreover, it was found through the uncertainty analysis that the principle was not sensitive to cost variance, although the margins do change.

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I. Introduction

THIS paper focuses on the development of an aircraft design optimization methodology that models uncertainty and sensitivity analysis in the tradeoff between manufacturing cost, structural requirements, and aircraft direct operating cost (DOC). The application considered is the design of fuselage stringer-skin panels, typically driven by a minimal-weight Goal Seek in Microsoft Excel. This involved the development of a methodology that facilitated the optimal choice of the associated design variables, in order to deliver minimal DOC, and is relative to the transportation of the airframe weight during the aircraft's life span. The optimization process considers the structural requirements, structural configuration, and manufacturing cost in generating a design that minimizes the cost to the airline operator, but it does not consider maintenance issues or nonrecurring costs.

A key contribution of the paper is to investigate the modeling of uncertainty and to perform a sensitivity analysis to investigate the robustness of the optimization methodology. Stochastic distributions are used to characterize manufacturing cost distributions, and Monte Carlo analysis is performed in modeling the impact of uncertainty on the cost modeling. The results are then used in a sensitivity analysis that incorporates the optimization methodology. In addition to investigating manufacturing cost variance, the sensitivity of the optimization to fuel burn cost and structural loading is also investigated.

II. Modeling and Optimization

A. Manufacturing Cost Modeling

The authors developed a method of modeling manufacturing cost that is referred to as the genetic-causal method. This is achieved by 1) classifying the generic cost elements that are linked to particular (genetic) design (product) attributes and 2) developing causal parametric relations that link those genetic attributes to the resultant manufacturing (or life cycle) costs. This results in scientifically based relations that numerically link cost to causal sources that are explicit to design definition. Apart from being a highly generic cost-modeling technique, the genetic-causal method is also well suited to use within an integrated design platform, as changes in the design (for performance benefit) can be mapped through to cost in order to directly tradeoff manufacturing cost relative to some global objective functions, as exemplified later in this paper.

The manufacturing cost modeling is based on empirical data gathered from Bombardier Aerospace Shorts and is typical for regional passenger jets. The genetic-causal approach imposes a typical initial breakdown of the cost into material cost, fabrication cost, and assembly cost. The generic product families used on a typical stringer-skin panel are the skin panel and the stringers and frames that support the skin in the longitudinal and lateral directions, respectively. In addition, there are cleats at every stringer-frame junction and rivets that are used to fasten the structures together. For these families, the overall breakdown in the manufacturing cost analysis is summarized through

$$C_{\text{panel}} = \sum_{i=1}^{5} C_i = C_{\text{skin}} + C_{\text{stringers}} + C_{\text{frames}} + C_{\text{cleats}} + C_{\text{rivets}} \quad (1)$$

where C_{panel} is the total cost of the panel and C_i the total cost for the family *i*. The three major types of costs to be taken into account (i.e., materials, fabrication, and assembly costs) are typically found to be 28, 37, and 35%, respectively, [1] for the panels under consideration. A typical aluminum panel is shown in Fig. 1 with sheet metal frames in the vertical plane and stingers running horizontally along the length of the skin, all fastened together using riveting and cleats (sheet metal pressed brackets). For each of the part families defined in Eq. (1), semi-empirical equations were established that included material cost C_i^m and labor cost C_i^l, the latter corresponding to either fabrication or assembly costs: $C_i = C_i^m + C_i^l$, where superscripts m and l refer to either material or labor, respectively. In addition, there is a material coefficient c_i^m ($/unit) and two labor coefficients: the time factor c_i^l (h/unit), which already includes such things as learning curve or breaks, and the wage rate per hour r_i^l ($/h).

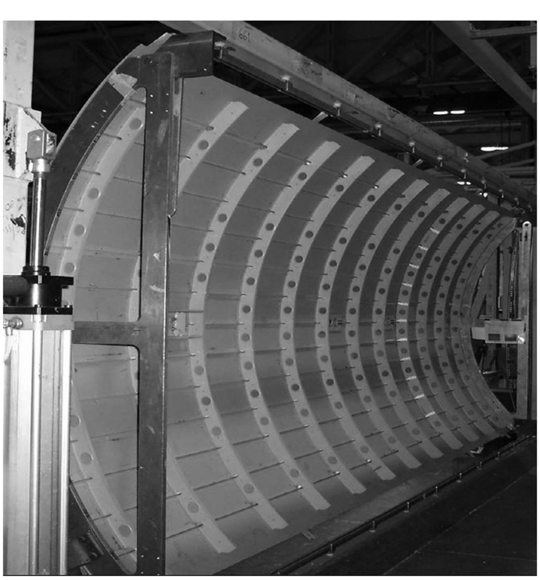

Fig. 1 Section of the panel.

If one takes the frames as an example, the material cost is computed as a function of the volume. The frame geometry is of a C-shaped cross section, and the length corresponds to the panel width W. If t_f is the frame thickness, h_f is the frame height, and l_f is the frame flange length, then the volume V_f of 1 C-shaped frame is given by

$$V_f = ((2l_f + h_f)t_f - 2(t_f)^2)W \quad (2)$$

Given the number of frames n_{frames}, the material density ρ, and the material cost coefficient c_{2024}^m (/g) for the 2024 T3 aluminum, the material cost for the frames is computed by

$$C_{\text{frames}}^m = n_{\text{frames}} V_f \rho c_{2024}^m \quad (3)$$

The frame labor coefficient c_{frames}^l (h/hole) is directly proportional to the number of lightening holes n_{holes}. If r_{frames}^l is the labor cost per hour (/h), the total labor cost for the frames can be calculated:

$$C_{\text{frames}}^l = n_{\text{frames}} n_{\text{holes}} r_{\text{frames}}^l c_{\text{frames}}^l \quad (4)$$

B. Optimization

The optimization is concerned with linking and trading off structural efficiency with manufacturing cost and both of their respective impacts on DOC. Structural efficiency is already a tradeoff between maximizing material strength utilization and reducing weight [1], and manufacturing cost is a tradeoff between specified design requirements and process capability [2]. Equation (5) highlights that the tradeoff can be achieved through the minimization of DOC:

$$\text{DOC} = fn\ (\text{ac}, \text{fb}, m, \text{cn}, \text{gs}) \quad (5)$$

where ac is acquisition, fb is fuel burn, m is maintenance, cn is crew and navigation, and gs is ground services.

However, for the purposes of the structural design tradeoff, the majority of these DOC cost drivers can be said to remain constant so that only acquisition cost and fuel burn are treated as design-dependent. The other elements are of little direct relevance to the structural airframe designer, apart from airframe maintenance, estimated by Sandoz [3] to be only 6% relative to the power plant and systems requirements. Acquisition cost is driven by the cost of financing the manufacturing the aircraft, with amortized nonrecurring elements included. However, the unit price can be stripped of its profit margin, overheads, and contingency to be seen as a function of the basic manufacturing costs in the design tradeoff. Fuel burn is a function of the specific fuel consumption and the cost of fuel and therefore can be said to be a function of weight in the current context. With regard to the performance-manufacture tradeoff, it has been noted by Rais-Rohani and Greenwood [4] that the minimum manufacturing cost does not necessarily reflect the minimum weight. Taking both acquisition cost (ac) and fuel burn cost (fbc) into account, a simple equation of the following form was used in the optimization to trade off the structural design solutions relative to the impact on manufacturing (mfc) cost and resulting DOC (doc):

$$\text{doc} = \text{fbc} + \text{ac} = \text{fbc} + n \cdot \text{mfc} \quad (6)$$

It can be seen from Eq. (6) that acquisition cost is expressed as some multiple n of manufacturing cost. This is introduced to allow the very basic manufacturing costs being modeled to be scaled up to reflect a more realistic DOC level impact (i.e., including all nonrecurring costs, factory and corporate overheads, profit, and financing). The pie chart shown in Fig. 2 shows that a 50% weighting for acquisition cost and 15% weighting for fuel burn is reasonable for the DOC split for an aircraft in the regional aircraft sector, in keeping with the panels addressed in the paper. However, it is reasonable to

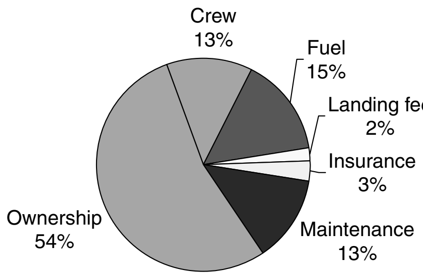

Fig. 2 DOC breakdown for a regional passenger jet [15].

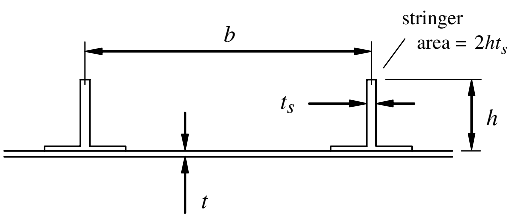

Fig. 3 Modeling of the panel for structural analysis.

assume that approximately 60% of the ownership cost is a function of the airframe, and the power plant, subsystems, and landing gear make up the remainder [5].

The savings in DOC that are directly attributable to the design of the panel are assumed to be made up of the savings in manufacturing cost, offset against a calculated cost penalty for any increase in structure weight. The cost penalty, at a fixed rate of \$300/kg, is based on the fuel burn of the typical operation of a civil transport aircraft expressed in terms of all-up weight (often referred to as the economic value of weight savings). The minimization of the total cost is the basis of the optimization and the structural analysis simply ensured that the panel continued to meet the applied loading requirement.

For this purpose, the fuselage panel was idealized, as shown in Fig. 3. A representative compressive loading intensity p was selected that, with a fixed frame pitch $L_f = 634$ mm, gave a structural index value $p/L_f = 0.5\ \mathrm{N/mm^2}$. This resulted in a relatively low stress level in the aluminum alloy panel, nevertheless appropriate to the design from which the actual cost data was extracted. Failure modes considered were long- and short-wave panel buckling, inter-rivet buckling (since riveting is a major cost factor), and material stress limits. Explicit formulas were derived for each of these to facilitate their use in combination with cost formulas in a cost–weight optimization, making use in part of data in the ESDU International Structures Series [6]. Further details of the structural analysis are given in the authors' previous paper [1], which used the same panel design.

A preliminary, purely structural, optimization of the panel (excluding cleats, rivets, and other secondary items) was first performed. Design variables are those shown in Fig. 3, together with the rivet pitch r_p. This gave a theoretical maximum structural efficiency $\eta = 0.693$ for the particular stringer type. The weight of this optimized panel and its cost according to the cost formulas of the previous section were then used as a baseline for decrease or increase in weight and cost in the subsequent work. Further optimization was performed for minimum total weight (i.e., now including secondary items), minimum bare material cost, minimum total manufacturing cost, and minimum DOC. For this purpose, the whole set of formulas derived for manufacturing cost and structural analysis was assembled in a Microsoft Excel spreadsheet. Optimization is performed by a reduced gradient method. This is very fast, as a result of the explicit formulation, as necessary for the subsequent work.

Tables 1 and 2 summarize changes in panel dimensions after optimization and savings achieved according to the choice of objective (cost savings are in U.S. $/\mathrm{m^2}$, weight is in $\mathrm{kg/m^2}$, and negative values indicate an increase from the theoretical maximum structural efficiency $\eta = 0.693$). Not surprisingly, optimizing for DOC rather than minimum weight reduces the number of stringers, with corresponding increase in skin and stringer thickness. This reduces both the number of cleats and the amount of riveting, and this is responsible for a major part of the cost saving. Full details of the procedure and the results obtained are given in the previous paper [1]. However, it might be noted that the actual savings achieved of $640/\mathrm{m^2}$ would amount to an estimated savings in DOC for a small regional jet aircraft of \$500,000 for the fuselage alone.

As already stated, the results referred to above are based on an acquisition cost expressed as some multiple n of manufacturing cost and a cost penalty related to fuel burn. A fixed value of n was chosen (typical values might be between 2 and 3.5) and a fixed fuel burn penalty. In the following section, the work is extended through an uncertainty analysis to investigate variation in manufacturing cost, and a sensitivity analysis is performed to examine the effect of the chosen value of n and the fuel burn penalty on the validity of the present approach and conclusions drawn from it.

III. Uncertainty Analysis

The objective of uncertainty analysis is to investigate likely variance in the data and errors in the cost modeling presented, as defined by regression analysis and the experimental data. This will be presented using a Monte Carlo analysis, which is based on the generation of multiple trials to determine the expected value of cost as the random variable. The initial step in Monte Carlo simulations is the generation of the random variables. Therefore, there follows a description of the Student's t-distribution, which is especially suited for estimation with small samples and for regression analysis. Then the technique proposed for the computation of the cost distribution using a Monte Carlo method is described, and finally, the results obtained are presented relative to the modeling and optimization set out in the previous section.

A. Required t-Distribution

The t-distribution is required for the generation of random variables, which will then be used in the Monte Carlo analysis. It is known that a continuous random variable Y only has a t-distribution if and only if its probability density function is given by [7]

$$f_t(y|\theta,\alpha,\nu) = c_t^{-1}\left[1+\frac{1}{\nu}\left(\frac{y-\theta}{\alpha}\right)^2\right]^{-(\nu+1)/2}, \qquad -\infty < y < \infty \quad (7)$$

where $-\infty < \theta < \infty$, $\alpha > 0$, $\nu > 0$, and the integrating constant is

$$c_t = c_t(\nu) = B\left(\frac{1}{2},\frac{1}{2}\nu\right)(\nu\alpha^2)^{1/2} = \frac{\Gamma(\frac{1}{2}\nu)\sqrt{\pi}}{\Gamma(\frac{\nu+1}{2})}(\nu\alpha^2)^{1/2} \qquad (8)$$

Table 1 Panel dimensions (in millimeters) after optimization

Objective[a]	η	b	h	t	t_s	r_p
Efficiency η	0.693	42.8	27.6	0.85	1.61	31.1
Minimum W	0.632	71.5	31.0	1.60[b]	1.60[b]	61.6
Minimum mat	0.628	65.5	27.0	1.09	2.53	41.9
Minimum mfc	0.383	192.3	38.64	2.52	6.07	124.7
Minimum doc	0.517	125.1	28.2	1.97	3.73	83.7

[a]W is the total weight, mat is the bare material cost, mfc is the total manufacturing cost, and doc is the direct operating cost.
[b]Values indicate that the limits of validity of the local buckling data have been reached.

Table 2 Savings (U.S. $/\mathrm{m^2}$) according to objective

	Savings in			
Objective[aa]	W	mat	mfc	doc
Minimum W	1.60	−11	807	2898
Minimum mat	0.99	36	680	2335
Minimum mfc	−2.29	−108	1186	2872
Minimum doc	0.58	−26	1122	3539

[a]W is the total weight, mat is the bare material cost, mfc is the total manufacturing cost, and doc is the direct operating cost.
[b]Values indicate that the limits of validity of the local buckling data have been reached.

where B is the beta function that can be decomposed in terms of the gamma function Γ as follows:

$$B(\beta, \gamma) = \frac{\Gamma(\beta)\Gamma(\gamma)}{\Gamma(\beta+\gamma)} \tag{9}$$

$$\Gamma(\beta) = \int_0^\infty t^{\beta-1} \exp(-t)\, \mathrm{d}t, \quad \text{with } \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi} \tag{10}$$

The standardized random variable $Z \equiv (Y-\theta)/\alpha$ has a probability density function of the form given above with $\theta = 0$ and $\alpha = 1$, known as Student's t-distribution and having ν degrees of freedom. As $\nu \to \infty$ the t-distribution approaches a normal distribution with mean θ and variance α^2. The t-distribution has a bell shape similar to the standard normal distribution but is more spread out (has a larger variance). Figure 4 represents the Student's t-distribution with four different values of ν as well as the standardized normal distribution.

For estimations with small sample sizes, it is also known that the Student's t-distribution should be used rather than the normal distribution [8]. In this case, only the sample mean and variance can be estimated, and so the confidence interval for estimating the population mean is based on the fact that the random variable has a t-distribution with ν degrees of freedom:

$$T = \frac{\bar{X} - \propto}{\sqrt{s^2/n}} \tag{11}$$

where n is sample size, $\bar{X}$ is sample mean, $\propto$ is population mean, and s is sample standard deviation. The number of degrees of freedom is given by the sample size minus one (i.e., $\nu = n - 1$). The sample mean $\bar{X}$ is an unbiased estimator for $\propto$ and serves as a point estimate for $\propto$. The α% confidence interval for the mean is given by

$$[\bar{X} - t_{\nu,\alpha}\sqrt{s^2/n}, \bar{X} + t_{\nu,\alpha}\sqrt{s^2/n}] \tag{12}$$

where $t_{\nu,\alpha}$ is the value of t that limits the extreme α% (or $\pm\alpha/2$% either side) of the t-distribution with ν degrees of freedom.

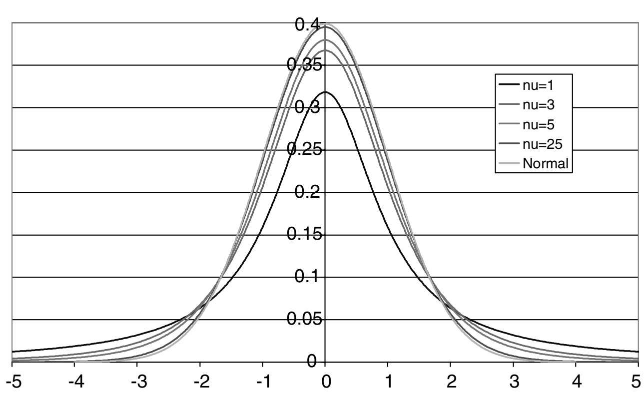

Fig. 4 Example of the Student's t-distribution.

In the case of regression, as used in the earlier cost modeling, the Student's t-distribution can be used to predict the confidence intervals for whatever sample size [8,9]. Considering a linear regression line of the form $y = Ax + B$, the confidence intervals for A and B can be computed using the sampling variances of A and B, s_A^2, and s_B^2, respectively. Detailed equations for the regression analysis and estimation of the variances can be found in Taylor [10]. These values can be automatically computed in Excel in an ANOVA table generated using the Analysis ToolPak add-ins. The α% confidence intervals for A and B, are given by

$$[A - t_{\nu,\alpha}s_A, A + t_{\nu,\alpha}s_A] \tag{13}$$

$$[B - t_{\nu,\alpha}s_B, B + t_{\nu,\alpha}s_B] \tag{14}$$

The number of degrees of freedom ν corresponds to the number of data points minus the number of coefficients in the regression line (i.e., $\nu = n - 2$). Similarly, if the regression line is of the form $y = Ax$, the α% confidence interval for A remains as in Eq. (13) although the number of degrees of freedom becomes $\nu = n - 1$.

In summary, a Student's t-distribution can be generated to represent the distribution of the coefficients of the cost equations that have been defined by regression analysis. However, this distribution is not directly available in Excel and needs to be generated. A traditional way to generate a Student's t-distribution is by combination of the normal and chi-squared distributions [11] or other alternatives such as the acceptance–rejection algorithm (TAR) used by Kinderman et al. [12]. The TAR algorithm was preferred by virtue of its efficiency and programming ease. It uses random variables U_i generated by the uniform distribution $u(0, 1)$, uniform on the interval [0, 1] (uniform distributions can be generated directly in Excel using the Analysis ToolPak add-ins).

B. Monte Carlo Simulation

There is commercial risk analysis software that can carry out Monte Carlo analysis. Nevertheless, the Monte Carlo method was coded explicitly into the Excel model in order to provide a more transparent understanding of the basic procedure and to facilitate flexibility in the optimization context. In terms of the development of the routine, the rates and utilization factors were treated as fixed data inputs, as these values do not have an observed statistical distribution, as evident with the cost of labor and equipment and associated with the performance of the company. The analysis of fixed data inputs is more appropriately investigated through sensitivity analysis, such as that described in the following section. The majority of input cost data such as labor hours or material costs have been modeled as being uncertain, and this is appropriately investigated with the Monte Carlo technique.

When sufficient statistical data are available, the statistical distributions are based on the actual mean and variance obtained by the regression analysis. Otherwise, when only one value is available, it is used as the mean, and the standard deviation (square root of the variance) is defined as a percentage of this value. On one hand, a Student's t-distribution is generated for each of the cost variables that are being characterized through regression analysis. The number of degrees of freedom ν corresponds to two less than the number of data

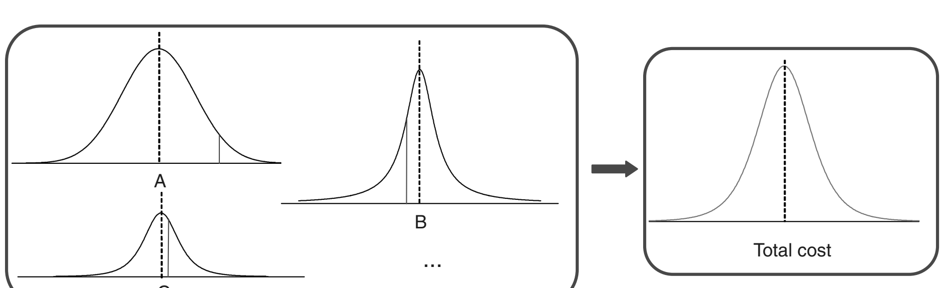

Fig. 5 Illustration of the Monte Carlo process.

Table 3 Data used to generate the Student's t-distributions

Variable	A	DRCL	MANR	DRSF	AUTR	C-T	D-T	ALS-T	BLS-T	C-Z	D-Z	ALS-Z	BLS-Z	ASS-Z	BSS-Z	MCF
ν	15	5	9	5	3	5	5	24	24	7	7	44	44	27	27	84
X	0.189	0.0185	0.0111	0.0142	0.0050	0.489	0.0083	0.187	39.60	0.347	0.0756	0.457	11.23	0.370	22.88	0.151
s	0.004	0.0015	0.0005	0.0008	0.0009	0.009	0.0023	0.034	6.29	0.060	0.0126	0.098	16.78	0.171	4.31	0.041
%	2.07%	8.23%	4.93%	5.55%	17.28%	1.86%	28.33%	18.05%	15.88%	17.34%	16.66%	21.40%	149.40%	46.13%	18.82%	27.42%

points used if the regression line was of the form $y = Ax + B$ (where A is the slope and B is the intercept) or one less if the regression line was of the form $y = Ax$. The distributions generated are normalized and centered around zero. The point estimates A and B and their variances s_A^2 and s_B^2 were then used to translate and scale the distribution to the appropriate values. These estimates and variances can be calculated using appropriated formulas [10] or by using the results of the analysis of variance (ANOVA) that is available in Excel. However, attention must also be paid to the fact that the various formulas do differ depending on the form of the regression equations. On the other hand, normal distributions are generated for the variables that have no statistical data regarding variance. The latter is the case with the material cost of a skin panel or the chemimilling cost coefficients.

The total manufacturing cost is computed using the randomly distributed values of the cost coefficients, as shown in Fig. 5, where A, B, and C represent the cost coefficients. As these values have been stored randomly, the program naturally picks data in all the distributions, choosing larger or smaller values at random and respecting the frequencies imposed by the distribution. The final cost is then also randomly distributed and statistical analysis can be performed on the results.

C. Generation of the Cost Distributions

The data used to generate the Student's t-distributions are presented in Table 3 for each coefficient that has been defined by regression analysis, where ν is the number of degrees of freedom, X is the point estimate, and s is the standard deviation. The last row indicates the percentage of the point estimate that the standard deviation represents. Two different sets of coefficients are proposed for the manufacturing cost of Z-stringers, one set for small interwindow stringers, and one set for large stringers.

Table 3 gives more precise information on the definition of the coefficients and on the units used. With regard to the analysis, the aim of Table 3 is to provide information on the dispersion of the regression coefficients in terms of percentage (last row). It can be seen that the majority of these values are less than 20%. The greatest variance appears for the coefficients BLS-Z (intercept of the manufacturing cost for large Z-stringers) and ASS-Z (slope of the manufacturing cost for small Z-stringers), as described in Table 4.

For illustration purposes, examples of samples generated using the Student's t-distribution are presented in Fig. 6 for two of the cost coefficients: histogram A displays a low percentage variation, and AUTR (automatic riveting) has medium variation. For each new simulation, these samples are generated using a different initial seed so that they are slightly different each time. These samples have been scaled proportionally so that their mean is 1. They can then be represented using the same axes and compared graphically as presented on Fig. 7, which shows that the distribution for AUTR is much wider than the distribution for A, as the bins were widened to accommodate AUTR.

The statistical analysis for the two samples of Figs. 6 and 7 is given in Table 5. It can be verified in Table 5 that each mean obtained is closed to the corresponding point estimate value presented in Table 3. The kurtosis indicates that the distributions sampled are flatter than a normal distribution, which is a characteristic of a Student's t-distribution, with the kurtosis of a normal distribution of 0. The last row indicates the percentage of variation compared to the mean $\propto$ for a 1σ analysis if the distribution had been normal, with the cost coefficient given by $\propto \cdot (1 \pm x\%)$, where x is the value indicated in the table. It can be verified that these percentages are of the same order of magnitude as those indicated in Table 3.

D. Computation of the Manufacturing Cost by Monte Carlo Analysis

The statistical results of the Monte Carlo simulations are presented in Tables 6 and 7 for the manufacturing cost of the AFTLH panel; the real costs have been altered for proprietary reasons but left at a realistic level to show typical magnitudes for each of the statistical parameters. The model has been developed by introducing progressively more uncertainties, either by introducing more variables with

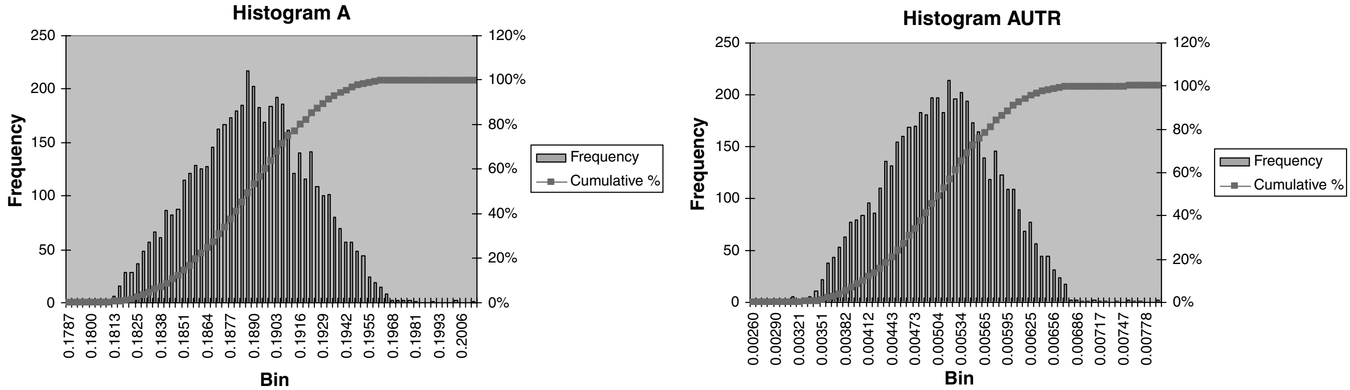

Fig. 6 Histograms for sheet metal (A) and AUTR for Student's *t*-distributions.

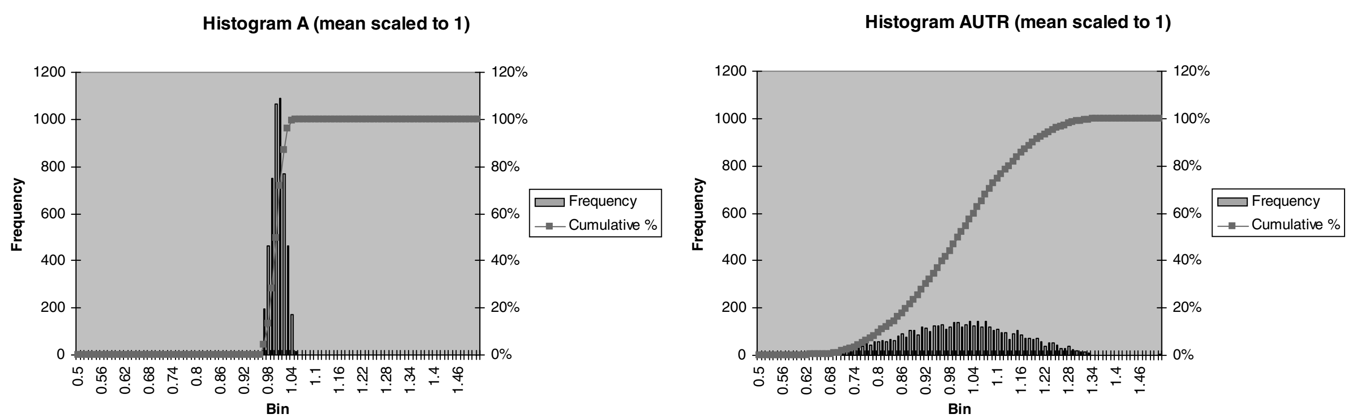

Fig. 7 Histograms for sheet metal (A) and AUTR with means scaled to 1.

uncertainties or by increasing the uncertainties for the variables generated by a normal distribution. During this process, it has been shown that the width of the distribution for the manufacturing cost, measured by the standard deviation, increases if the uncertainty in the input parameters increases, which is evident (see, for example, simu 1, simu 2, simu 3-4-5, and simu 7 in Table 6). The first simulations had uncertainties for only the coefficients that were determined using regression analysis with the Student's *t*-distributions, and uncertainties on the other parameters were introduced later (using normal distributions). The standard deviation for the coefficients for which the distribution is given by a normal distribution (those for which no statistical experimental data were available) has been varied from 5 to 10% between simu 2 and simu 3 and from 10 to 20% between simu 3 and simu 7. Each simulation generates a chosen number of outputs, which form the sample to be analyzed statistically. Generally, the number of outputs was 1000 or 5000, depending on the simulation. The minimum sample size that has to be generated in a Monte Carlo analysis to obtain a sample mean with a given percentage of accuracy can be determined theoretically. Knowing that, the sample size can be reduced to limit computing time. Nevertheless, the simulations performed here were not time–consuming and a sample size of either 1000 or 5000 was acceptable. Comparing simu 3-4-5 with simu 6 in Tables 6 and 7, it can be seen that reducing the sample size or number of outputs leads to an increase in the spread of the distribution.

The same simulations have been run several times and the results obtained in terms of mean, standard deviation, and standard error have been compared, as well as the histograms (compare simu 3-4-5 in Tables 6 and 7). The standard deviation represents the deviation of the sample itself (i.e., the square root of the variance of the sample), and the standard error represents the deviation of the mean as stated by the central limit theorem [i.e., the square root of the sample

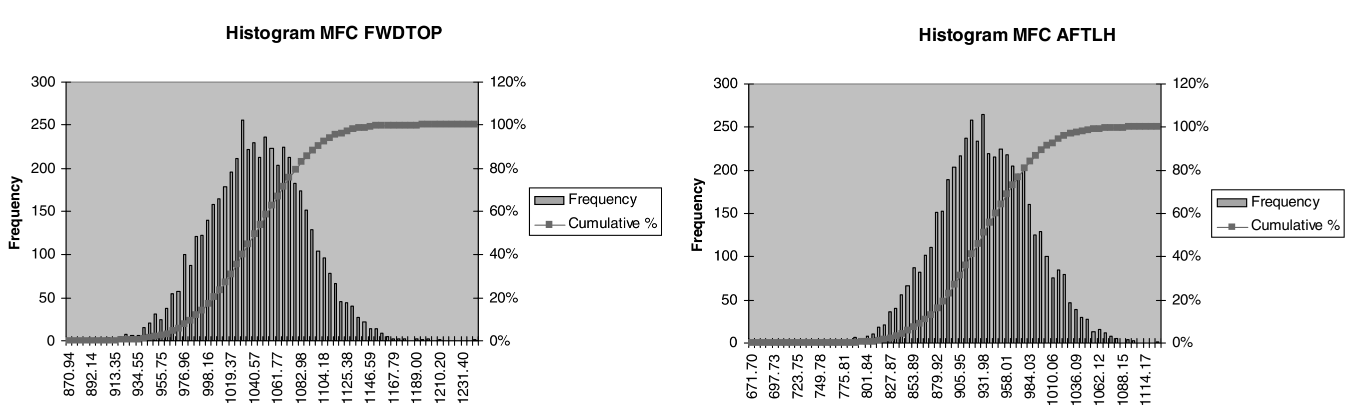

Fig. 8 Histograms for the manufacturing cost (MFC) ($/m^2$) of two panels.

Table 4 Definitions and units for the cost coefficients

Coefficient	Definition	Units
A	Sheet cost coefficient	£/in.^{3}
DRCL	Set up and drilling for cleats	h/hole
MANR	Manual riveting	h/rivet
DRSF	Set up and drilling for frames and stringers	h/hole
AUTR	Automatic riveting	h/rivet
C-T	Slope of material cost coefficient for T-stringers	£/in.^{3}
D-T	Intercept of material cost coefficient for T-stringers	£/in.
ALS-T	Slope of manufacturing cost coefficient for T-stringers	h/in.^{3}
BLS-T	Intercept of manufacturing cost coefficient for T-stringers	h/in.^{2}
C-Z	Slope of material cost coefficient for Z-stringers	£/in.^{3}
D-Z	Intercept of material cost coefficient for Z-stringers	£/in.
ALS-Z	Slope of manufacturing cost coefficient for long Z-stringers	h/in.^{3}
BLS-Z	Intercept of manufacturing cost coefficient for long Z-stringers	h/in.^{2}
ASS-Z	Slope of manufacturing cost coefficient for small Z-stringers	h/in.^{3}
BSS-Z	Intercept of manufacturing cost coefficient for small Z-stringers	h/in.^{2}
MCF	Make cost for frames	h/hole

Table 5 Statistical analysis of Student's t-distribution

Variable	A	AUTR
Mean, m	0.1887	0.00504
Standard error	0.000046	0.0000103
Median	0.1888	0.00505
Standard deviation σ	0.0032	0.00073
Sample variance	0.000010	0.0000005
Kurtosis	−0.4257	−0.3333
Skewness	−0.0075	−0.0008
Range	0.0226	0.00533
Minimum	0.1787	0.00260
Maximum	0.2013	0.00793
Count	5000	5000
$1\sigma \pm$ %	1.71	14.50

variance divided by the sample size ($\sqrt{\sigma^2/n}$)]. In practice, from a statistical point of view, a single generated sample would give enough information and there is no need to run the simulation several times, apart from the current purpose of verification and comparison. As it is only possible to generate a pseudorandom number, an initial seed has to be given as the input to each new simulation; otherwise, the same scheme would be reproduced for each simulation. The results are evidently different between the three simulations but still consistent.

The kurtosis and skewness of the distribution were also analyzed; these values were 0 for the normal distribution. The kurtosis is a relative measure of the shape as compared to that of the normal distribution. A negative kurtosis denotes that the distribution is flatter than the normal distribution, and a positive kurtosis denotes the contrary. The skewness is a measure of asymmetry; a negative skewness indicates that the tail of the distribution extends toward the left, and a positive skewness indicates an extension toward the right. Table 5 can be checked to see that the kurtosis and skewness of the distributions obtained by Monte Carlo analyses for each the seven simulations are close enough to zero to consider these distributions as normal when applying classical statistical analysis tools and tables. This is in agreement with the central limit theorem and its extension to large samples.

For a normal distribution, the standard deviation σ corresponds to a 68% confidence limit. It is possible to increase the percentage of confidence by increasing the confidence interval. This will give more probable but less precise answers, with 4σ corresponding to a 99.99% confidence. Table 8 indicates the percentage of confidence according to the number of σ considered in the analysis [10]. Table 7 summarizes the confidence intervals for the manufacturing cost for the seven simulations. The first set of results (1σ) indicates that there is a probability of 68% that the manufacturing cost is between the minimum and maximum values, and the second set of results (4σ) indicates that there is a probability of 99.99% that the manufacturing cost is between the corresponding minimum and maximum values. The percentage of variation compared to the mean $\propto$ is also indicated for both cases; the manufacturing cost is given by $\propto \cdot (1 \pm x\%)$, where x is the value indicated in the table.

The evolution of the results in Table 7 follows the same logic of Table 6, although presented in a more concise manner. It can be seen that increasing the uncertainty or reducing the sample size increases the percentage deviation. The order of magnitude of the maximum deviation is 6% for a 1σ analysis and 23% for a 4σ analysis, with a 10% variation for the coefficients for which only one value was available. The magnitudes increase up to 7 and 28%, respectively, if a 20% variation is considered for these coefficients, which are having more influence on the final variation in this case. Nevertheless, as these coefficients have been provided in this form by the industrial partner, they can be considered as correct, and therefore a 10% of variation is appropriate.

Tables 9 and 10 present the results of the Monte Carlo analysis for the manufacturing cost of six panels that are used across three aircraft deviants, as studied. For these simulations, the sample size is 5000, uncertainties are given for all the cost coefficients, and the standard deviation for the generation of each normal distribution is 10% of the mean. The actual value of each panel, adjusted for proprietary reasons, is also given in Table 9, as well as the percentage of error between the expected manufacturing cost obtained with the cost model and the actual cost. It can be seen from comparison of the two tables that the actual value falls between the minimum and maximum values given by the 1σ analysis for four of the panels. For these panels, the error between the mean and the actual value is less than 4%. For the two remaining panels, the actual value falls between the limits values of the 4σ analysis, and the errors between the mean and the actual value are more important. This is not

Table 6 Statistical results for the manufacturing cost (/m^2) of the AFTLH panel

Variable	Simu 1	Simu 2	Simu 3	Simu 4	Simu 5	Simu 6	Simu 7
Mean $\propto$	933.25	934.04	933.32	932.46	931.78	935.21	935.86
Standard error	0.71	0.72	0.76	0.76	0.74	1.72	0.93
Median	932.23	933.59	933.69	932.60	930.63	936.09	936.18
Standard deviation σ	50.09	51.09	53.58	53.82	52.62	54.26	65.84
Sample variance	2509.33	2610.44	2871.27	2896.93	2768.42	2944.44	4335.02
Kurtosis	−0.1958	−0.1256	−0.0477	−0.2085	−0.0374	−0.2370	−0.0918
Skewness	0.0723	0.0287	−0.0023	0.0383	0.0245	0.0386	−0.0438
Range	346.68	331.13	379.38	395.81	455.49	318.78	491.24
Minimum	752.66	765.47	732.53	747.61	671.70	781.81	665.80
Maximum	1099.33	1096.60	1111.91	1143.41	1127.19	1100.59	1157.04
Count	5000	5000	5000	5000	5000	1000	5000
Number of uncertain variables	12	25	25	25	25	25	25
Deviation (normal distrib)	——	5%	10%	10%	10%	10%	20%

Table 7 Confidence intervals for the manufacturing cost ($/m^2$) of the AFTLH panel

Result	Simu 1	Simu 2	Simu 3	Simu 4	Simu 5	Simu 6	Simu 7
			1σ				
Minimum	883.15	882.95	879.73	878.64	879.16	880.95	870.02
Maximum	983.34	985.13	986.90	986.28	984.39	989.48	1001.70
Range	100.19	102.19	107.17	107.65	105.23	108.53	131.68
±%	5.37	5.47	5.74	5.77	5.65	5.80	7.04
			4σ				
Minimum	732.88	729.67	718.98	717.17	721.31	718.16	672.50
Maximum	1133.62	1138.41	1147.66	1147.75	1142.24	1152.27	1199.22
Range	400.75	408.74	428.67	430.59	420.93	434.10	526.73
±%	21.47	21.88	22.96	23.08	22.58	23.2	28.14

Table 8 Probability that a random variable will fall within k standard deviations of its true value

k	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2.0	2.5	3.0	3.5	4.0
Probability %	0	20	38	55	68	79	87	92	95.4	98.8	99.7	99.95	99.99

unreasonable, as similar conclusions in terms of errors between the cost model and the actual values had been found in previous work by Castagne et al. [13].

Another interesting conclusion relating to Tables 9 and 10 is that the deviations for the crown panels (FWDTOP, MIDTOP, and AFTTOP) are smaller than those of the side panels (FWDLH, MIDLH, and AFTLH). This is due to the fact that the data for the Z-stringers that are used for the side panels have larger dispersions than the data for the T-stringers that are used for the crown panels, as seen in Table 3. The maximum deviation of the results can then be reduced to 17% of the mean for a 4σ analysis of the T-stringer panel family. Figure 8 presents, for illustration purposes, histograms of the manufacturing cost for the forward top and left-hand-side panels.

In summary, it has been shown that the variations in manufacturing cost due to variations in the experimental data were typically less than 5% for a 1σ analysis and less than 20% for a 4σ analysis, with a 10% variation for the data when only one point value was available. This is significant, as even if the regression analyses had a poor statistical significance in terms of adjusted R-squared values for some of the cost coefficients, the deviations for the final cost results remain acceptable, taking into account the variations in the aforementioned inputs.

IV. Sensitivity Analysis

The previous section highlighted that uncertainty does exist. It is often assumed that this is especially true of cost, although cost is a particularly stringent customer requirement and perhaps variations in performance predictions are not often highlighted for the final product. Nevertheless, the variations in cost can be very large, and therefore the uncertainty analysis considered variations relating to a standard deviation σ corresponding to a 68% confidence limit, and even 4σ, with a probability of 99.99% that the manufacturing cost is between the corresponding minimum and maximum values. Such large variations may arise due to new processes, company efficiency, material costs, labor rates, design definition, etc. [14]. It was shown in Eqs. (5) and (6) that DOC is a function of both fuel burn costs and manufacturing costs: $\mathrm{doc} = \mathrm{fbc} + \mathrm{ac} = \mathrm{fbc} + n \cdot \mathrm{mfc}$. Consequently, the manufacturing factor n can be used to investigate the sensitivity of the system to variations in manufacturing cost. Although

Table 9 Statistical results for the manufacturing cost ($/m^2$) of the six panels

Result	FWLH	FWDTOP	MIDLH	MIDTOP	AFTLH	AFTTOP
Mean $\propto$	861.10	1041.28	715.66	825.05	931.78	1030.16
Standard error	0.67	0.63	0.59	0.49	0.74	0.59
Median	860.18	1041.31	714.80	825.44	930.63	1029.95
Standard deviation σ	47.08	44.56	41.91	34.42	52.62	41.56
Sample variance	2216.83	1985.47	1756.24	1184.84	2768.42	1727.02
Kurtosis	−0.1802	0.0492	−0.1868	0.0848	−0.0374	−0.0821
Skewness	0.0171	−0.0030	0.0145	−0.0144	0.0245	−0.0042
Range	345.54	371.07	286.65	283.69	455.49	307.12
Minimum	683.68	870.94	580.32	694.11	671.70	883.85
Maximum	1029.22	1242.01	866.97	977.81	1127.19	1190.98
Actual	896.91	1022.55	724.10	979.25	1008.50	1058.56
Error (mean versus actual)	−3.99%	1.83%	−1.17%	−15.75%	−7.61%	−2.68%

Table 10 Confidence intervals for the manufacturing cost ($/m^2$) of the six panels

Result	FWLH	FWDTOP	MIDLH	MIDTOP	AFTLH	AFTTOP
			1σ			
Minimum	814.02	996.72	673.75	790.63	879.16	988.60
Maximum	908.18	1085.84	757.57	859.48	984.39	1071.72
Range	94.17	89.12	83.82	68.84	105.23	83.11
±%	5.47	4.28	5.86	4.17	5.65	4.03
			4σ			
Minimum	672.77	863.04	548.03	687.37	721.31	863.93
Maximum	1049.43	1219.51	883.29	962.74	1142.24	1196.39
Range	376.67	356.47	335.26	275.37	420.93	332.46
±%	21.8712	17.1169	23.4231	16.6881	22.5873	16.1363

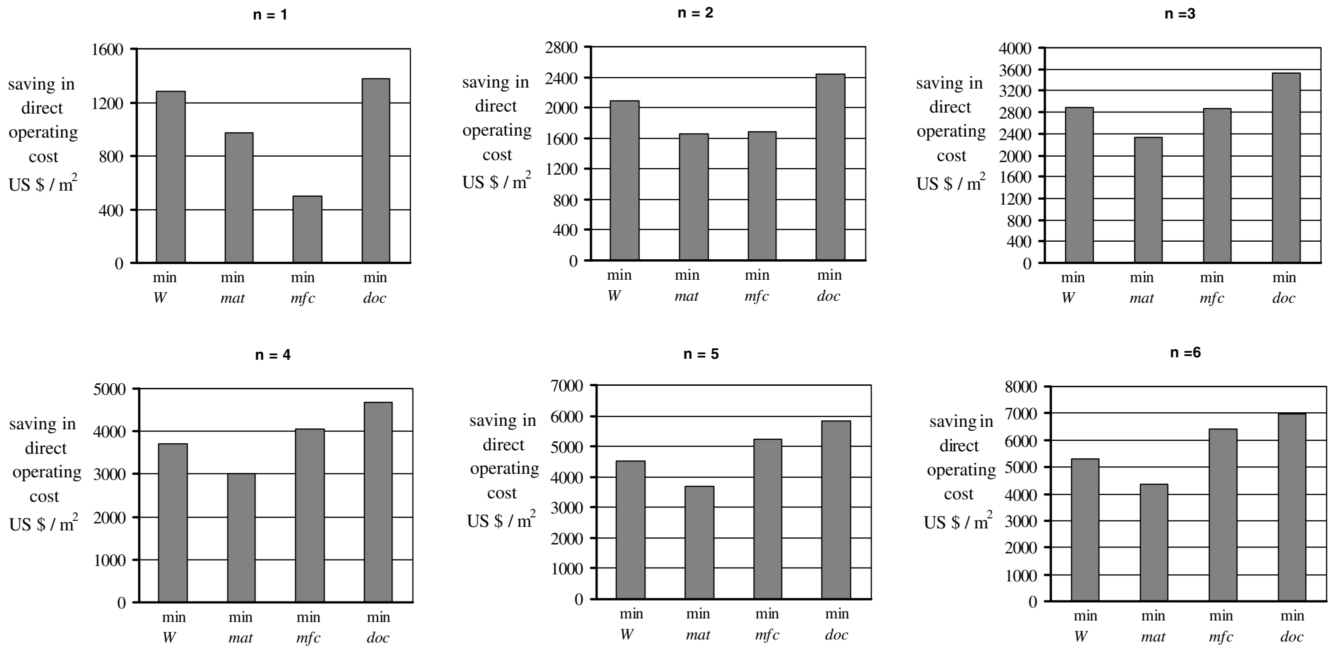

Fig. 9 Savings in DOC according to the choice of the objective function for different values of the multiplication factor *n*.

there is a focus here on modeling the sensitivity arising from uncertainty, it is also a simple matter to look at variations in fuel cost, structural load intensity, and the DOC penalty. Therefore, the sensitivity to manufacturing cost is first investigated in Sec. IV A, and the additional factors are presented in Sec. IV B. Typically, for the latter, variations in loading are of interest in terms of structural uncertainties or the generic application of the work across a wider range of aircraft, and variations in fuel cost are linked to the rising cost of fossil fuels and also the potential design impact of environmental levies and pollution penalties. In the analysis, the top panel used in the previous section is the applied case, with an applied structural load of 316.865 N/mm and a fuel cost of 300/kg, as in Secs. II and III.

A. Sensitivity Analysis of Manufacturing Cost

Figure 9 further supports the results of Sec. II in showing that focusing on minimizing DOC is significantly more rewarding than other traditional methods such as weight reduction.

It can be seen that the minimum manufacturing cost savings is equal to or higher than the minimum weight condition when n is greater than 3. When n becomes greater than 5, the geometric configuration does not change further when optimizing for DOC, and therefore the minimum weight result is not very different from that of $n = 5$. Prior to that, a significant gain in the minimal-weight condition had been evident in Fig. 9. For the value $n = 5$, the ratio between the fuel burn cost and acquisition cost is 3.3 when considering the reference optimization for maximum efficiency. This is close to the ratio given by the pie chart in Fig. 2 for total aircraft DOC, for which this ratio is 3.6. Therefore, considering that the influence of the panel can be extrapolated to the total aircraft, it can be said that a factor $n = 5$ is realistic.

Figure 10 represents the percentage of further gain in DOC that can be obtained when optimizing for minimal DOC rather than for minimal weight. It is computed by dividing the difference between the DOC for minimal DOC and the DOC for minimal weight by the DOC for minimal weight. This gain increases with n. For $n > 5$, the increase in the gain is proportionally less important (due to there being no change in geometric configuration).

B. Sensitivity Analysis for Fuel Cost and Applied Load

Investigating the fuel cost and applied load is important in terms of testing whether there are local optima and also if the initial results are applicable across a range of scenarios. The analysis used the same data as in the previous analysis with $n = 5$.

Figures 11–14 represent the optimal weight or DOC value as a function of fuel cost and load. All of the graphs are linear apart from Fig. 11. It can be seen at a glance that the optimization goal leads to reduced values of that quantity; that is, the values for weight in Fig. 13 are lower than those of Fig. 11 and vice versa for the DOC in

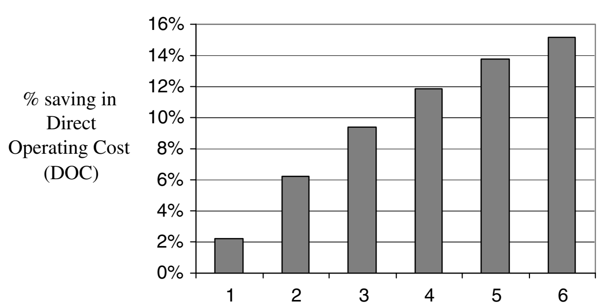

Fig. 10 Percentage of further gain in DOC obtained when optimizing for minimal DOC.

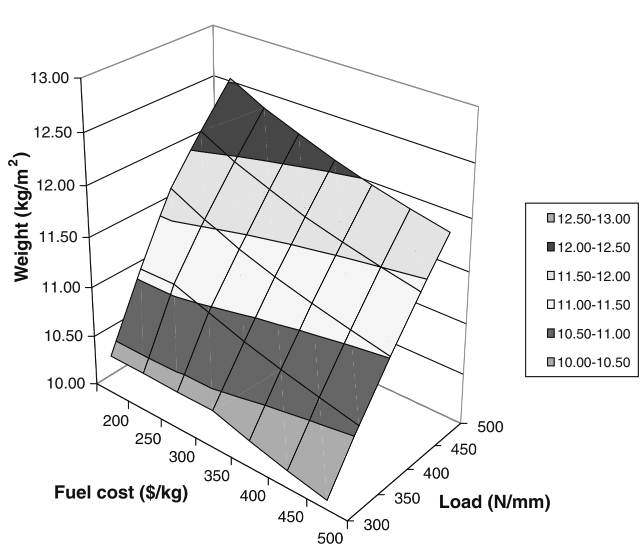

Fig. 11 Optimal weight as a function of fuel and load for minimal DOC.

Figs. 12 and 14. In general, Fig. 11 shows that weight is driven down with increasing fuel burn cost in order to minimize the increase in DOC shown in Fig. 14, whereas weight must be increased with higher applied loads (Fig. 13), which leads to higher DOC (Fig. 14). In Fig. 11, it can be seen that under a threshold value for the fuel cost, the configuration does not change and the weight is constant, which means that the fuel does not have any influence on the DOC. This threshold is higher for lower load and is detectable on the graph for a load of 300 and 350 N/mm. On the other hand, in Figure 13, the optimal weight has a discrete value for each load level that is not influenced by fuel cost. However, one of the key findings is that the DOC is much more sensitive to an increase in fuel burn than an increase in applied load. For example, in Figs. 13 and 14, an increase in load of 60% results in an increase in DOC of less than 10%, whereas a similar percentage increase in fuel burn results in a 20% increase in DOC. Therefore, it is concluded that costs are actually key engineering design drivers and should be equally, if not more, influential in the design process. This is fully consistent with the systems engineering approach in which customer requirements are made to drive the design synthesis.

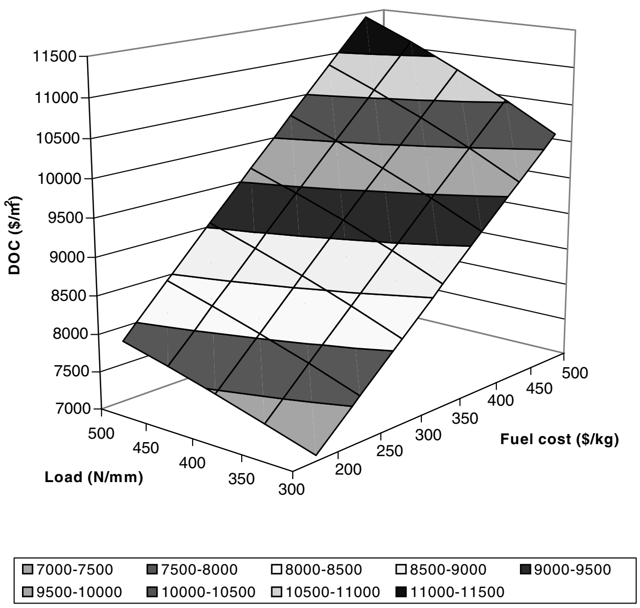

Fig. 12 DOC as a function of fuel and load when optimizing for minimal DOC.

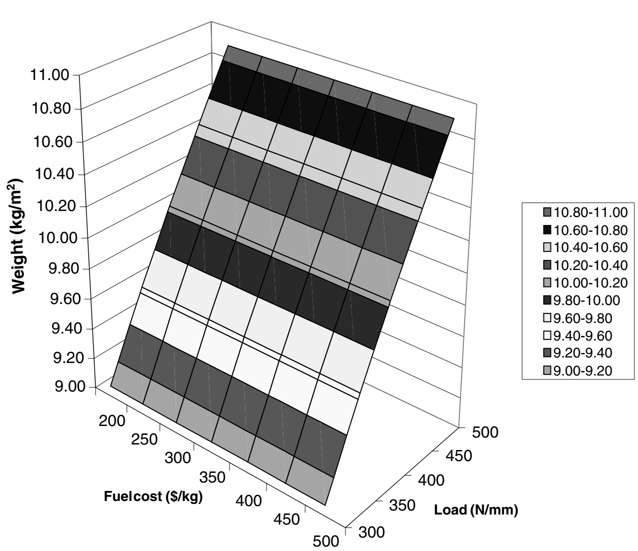

Fig. 13 Weight as a function of fuel and load when optimizing for minimal weight.

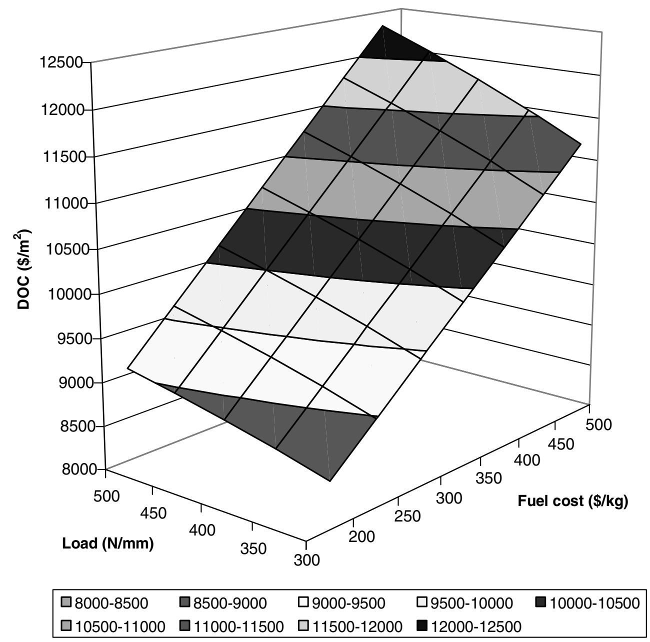

Fig. 14 DOC as a function of fuel and load when optimizing for minimal weight.

V. Conclusions

It has been shown that the design optimization process can be achieved by linking manufacturing costs models with structural models through an optimization analysis procedure driven by shared design parameters. The design-oriented causal basis for the assessment of manufacturing cost can be applied to the assessment of new design approaches and manufacturing processes, rather than assuming that the minimal-weight approach can be used to address the influence of DOC. However, the importance of addressing uncertainty and sensitivity is presented in the analysis, as costs and operating boundaries are not fixed and are subject to change and must be considered in the development of a robust cost-optimization process.

It has been shown that the minimal-weight condition does not result in minimal DOC, as it does not include the impact of manufacturing cost on acquisition cost; rather, there must be a tradeoff between minimizing weight and reducing manufacturing cost. This is an attractive finding for both the producer and the operator, as each see a direct financial return: the price of the aircraft is reduced to make the producer more competitive and the customer gaining early through reduced acquisition cost. However, these variables and parameters are subject to variance due to both uncertainty in the modeling of cost and changes in the design parameters. The uncertainty and sensitivity have been modeled and it has been shown that the basic finding of the research is true across a larger design space and for a range of cost variance: optimizing for DOC and considering manufacturing cost will result in a lower DOC than for the minimal-weight condition. However, it was also shown that at lower applied loads there is a threshold fuel burn cost at which the optimization process wishes to reduce weight, and this threshold decreases with increasing load.

A DOC savings of $640/\mathrm{m}^2$ of fuselage skin was evident, which relates to a rough order-of-magnitude savings of $500,000 for the fuselage (alone) for small regional aircraft. Moreover, it was found through the uncertainty analysis that the basic finding of the optimization principle was not sensitive to cost variance, although the margins were influenced.

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